# Longevity Index Technical Document

*(version 1.0)*

Description of the construction of various country-specific longevity indices produced by the **LLMA**

(Life & Longevity Markets Association)

19 March 2012



www.llma.org

# Contents

# 1. Background to LLMA Longevity Indices

### *1.1 Introduction*

The Life and Longevity Markets Association ('LLMA') is a non-profit organisation founded and funded by members, these being Aviva, AXA, Deutsche Bank, J.P. Morgan, Munich Re, Legal & General, Morgan Stanley, Pension Corporation, Prudential PLC, RBS, Swiss Re and UBS. The LLMA ('we') aims to promote the development of a liquid traded market in longevity and mortality-related risk. The association supports the development of consistent standards, methodologies and benchmarks to help build a liquid trading market of the type that exists for Insurance Linked Securities ('ILS'), and other large trend risks like interest rate and inflation.

The LLMA is launching a number of country-specific longevity indices on 19 March 2012 on the LLMA website (www.llma.org) to assist the development of such a market:

- England & Wales
- Germany
- Netherlands
- United States of America

The LLMA has produced this Technical Document to describe the construction of these longevity indices. A historical database of longevity indices will be published for these countries or country groupings, with further indices being produced on an annual basis as and when final data on both population exposures and deaths are available.

### *1.2 Guiding principles to longevity index development*

We define a longevity index as a body of data relating to the mortality, survivorship and life expectancy of a specified group of individuals, calculated according to robust and well-defined algorithms and processes.

It is useful to clarify the difference between a rate index (i.e. an index of observed rates) and a price index (i.e. an index of observed prices of securities or derivatives). Most indices in the financial world, e.g. the S&P 500 or the MSCI family of indices, are of the latter kind. An example of the former kind is Libor, which is an interest rate index. A longevity index is also an index of the former kind, i.e. a rate index of observed mortality rates, survival rates, life expectancies, etc. In the future when greater liquidity and transparency develops in the longevity market, it may be possible to develop longevity price indices that reflect the prices of longevity bonds or swaps.

The LLMA holds the view that there are a number of guiding principles that should guide the development and operation of any successful longevity index. These guiding principles are stated below:

*Tradability*

The index must be tradable, which means it needs to be credible, robust and have broad buy-in from market participants.

*Transparency*

Full Disclosure: Full and detailed disclosure of data sources, methodology, algorithms, rules, degree of discretion and governance procedures:

- Historical Data: Easy availability of historical data
- Clear Rules: Rules for dealing with missing data or late data need to be clear
- Minimal Scope for Discretion: Discretionary scope must be limited where possible and discretionary decisions concerning the index should be based on objective analysis and communicated to the market in a full and timely fashion

*Robustness*

The index needs robust procedures and processes for:

- Data collection
- Data validation
- Calculation rules and algorithms
- Production
- Publication

*Objectivity*

The index must be perceived to be objective, so that governance, ownership and production must be:

- Independent
- Impartial
- Free from conflicts

*Simplicity*

Simplicity should guide the design of an index as this aids transparency and robustness, and helps liquidity.

*Clear Governance*

Ownership of the index should be distinguished from its oversight and production.
Oversight of index should involve an appropriately staffed committee, which should have broad representation, ideally including external experts, and a mandate to maintain an index that encourages trading and liquidity.
Production of the index should be conducted by an independent "Index Calculation Agent".
Access to index data prior to publication needs to be controlled, in order to prevent individuals and/or institutions having privileged access to data.

*Timeliness*

The index must be updated regularly, with a frequency appropriate to the market.

*Continuity*

Ongoing production of the index must be ensured, and sub-indices (e.g. breakdowns by age, gender, etc.) should be stable and reliable.

*Consistency*

There should be consistency (i) in the methodology of construction of the index and (ii) in the provision of data (timeliness, process and data quality).

Universality

So that the longevity market grows as broadly and as consistently as possible, the index methodology should be widely applicable to creating indices for:

- Different countries
- Other broad-based populations of lives, such as industry or occupational groupings
- Customised pools of lives, e.g. for a large pension plan or group of pension plans
- The index should be relevant for pension plans, annuity portfolios, life portfolios, equity release mortgage portfolios, investors, etc.

## *1.3 Index Framework*

The three crucial elements that determine the nature of any longevity index are the underlying population, the body of mortality data associated with that population and the index methodology.

### *1.3.1 The underlying population*

The precise population used in the longevity index must be defined. This may be a closed population, e.g. the current set of members of a particular pension plan, or an open population, e.g. the national population of a country, which is open by virtue of migration.

### *1.3.2 The data*

The data used to calculate the index comprises exposure data (i.e. numbers of people by age, gender, etc.) and death data (i.e. numbers of deaths by age, gender, etc.) for the population underlying the index. The exposure data relate to a specific point in time, such as the beginning of a calendar year, or the mid-point in a calendar year. The death data relate to the number of deaths in a specific time period, which is likely to be one calendar year (1 January to 31 December), although shorter periods (e.g. quarterly) and longer periods may also be implemented if there is a perceived need.

The data need to be obtained from a reliable source, which provides as much confidence as possible in terms of the consistency of timing of data releases, the process of data collection and publication and the quality of the data.

### *1.3.3 The index methodology*

Whatever the choice of the index population and the source of the mortality data, the index methodology should be general enough that it can be applied to different populations. This facilitates standardisation and maximises familiarity among market participants when it comes to indices for different populations. Where it can be avoided, index methodology should not be *overly* tailored to the specific characteristics of a particular population or data set, although it must be acknowledged that some customisation may be desirable or necessary in particular cases.

### *1.3.4 Different metrics for mortality*

Crude mortality rates generally indicate that the calculation has been run directly on the numbers of deaths and population estimates as published, before any smoothing, averaging or adjustments have taken place. Two key mortality metrics that are often used are the central rate of mortality and the initial rate of mortality.

The central rate of mortality, $m_x$, is calculated as the numbers of deaths over a calendar year that were aged x years at their last birthday, divided by the mid-year population estimate (average exposure to risk over the year) for age x.

The initial rate of mortality, $q_x$, is the probability that a person aged exactly x years dies within the next year. For a closed population this would be calculated as the numbers of deaths over a calendar year aged x divided by the number of lives at the start of the year. For an open population it might be appropriate to make allowance for the effect of migration.

If we assume that deaths occur uniformly over the calendar year, the following approximation is widely used to link $q_x$ and $m_x$:

$$q_x = m_x / (1 + 0.5\, m_x)$$

### *1.4 Development of Lifemetrics*

J.P. Morgan LifeMetrics ('Lifemetrics') is a toolkit for measuring and managing longevity and mortality risk, designed by J.P. Morgan for pension plans, sponsors, insurers, reinsurers and investors. Lifemetrics was launched in 13 March 2007 and enables these risks to be measured in a standardised manner, aggregated across different risk sources and transferred to other parties

There are three components to Lifemetrics: Index, Framework & Software. Of relevance to this Technical Manual, the Lifemetrics Index contains current and historical longevity indices for England & Wales, Germany, Netherlands and the USA. These longevity indices contain separate data in respect of males and females, and provide the following types of metrics for different ages in specified years:

- Crude central rates of mortality
- Graduated initial rates of mortality
- Period curtate life expectancies

### *1.5 Transfer of Lifemetrics IP from J.P. Morgan to LLMA*

The LLMA members have been sharing intellectual property to advance the aims of the LLMA since the founding of the LLMA in February 2010.

On 26 April 2011, J.P. Morgan and the LLMA jointly announced that the intellectual property underlying the Lifemetrics Index would be transferred to the LLMA. This represented the culmination of four years of investment by J.P. Morgan and will now be made available through the LLMA for the benefit of all market participants.

The LLMA has combined the existing Lifemetrics technology with their own development work to launch the country-specific longevity indices described in this Technical Manual.

### *1.6 Index Oversight Committee and Independent Calculation Agent*

In accordance with the principle of clear governance, the LLMA has appointed an Index Oversight Committee and an Independent Calculation Agent.

The Independent Calculation Agent ('ICA') will produce all future longevity indices according to the methodology described in this Technical Manual, subject to any amendments that might be recommended by the Index Oversight Committee and subsequently approved by the Technical Committee of the LLMA ('Technical Committee').

The Index Oversight Committee ('IOC') has five current members and will meet on a quarterly basis to review any developments relating to data sources or index methodology. A representative from the Technical Committee will chair any meeting of the IOC. All IOC members should have recognised expertise in the development of tables or indices relating to either mortality or longevity and considerable practical experience of mortality analyses, whether in respect of national populations or subpopulations.

Those involved in the work of the ICA or members of the IOC should not be employed by one of the primary members of the LLMA. If an existing IOC member becomes employed by one of the primary members of the LLMA, or the employer of an existing IOC member becomes a primary member of the LLMA, the IOC member should step down from their position and the process of recruiting a new IOC member will begin.

For any current longevity index, situations will occasionally arise that require modification of the published calculation methodology. Whilst the LLMA seeks to minimise such situations, the LLMA proposes that the IOC has responsibility for recommending modifications to the calculation methodologies for country-specific longevity indices when necessary.

This would follow a prior escalation by the ICA or one of the primary members of the LLMA in response to either missing or problematic data, or real features in the data for which the current calculation methodology of a country-specific longevity index gave inappropriate or misleading results.

A recommendation would be expected when continued use of the existing calculation rules would lead to a material misstatement of the true state of the index and/or misrepresentation of the underlying mortality situation.

Any recommendation should be guided by the index principles and motivated by a desire to minimise market disruption and encourage the continued development of the market.

Sections 2 to 5 describe the underlying data sources that have been used to construct the country-specific longevity indices for those indices that are being launched on 19 March 2012.

# 2. LLMA longevity index for England & Wales

### *2.1 Source of population exposures and death data*

All-cause and cause-specific mortality statistics have been collected for England & Wales since 1841. Prior to 1959 death data was aggregated over 5-year or 10-year periods, with data in electronic format starting in 1901. Censuses are carried out every 10 years with the most recent census taking place in 2011. Intercensal and postcensal population estimates are produced by the Population Estimates Unit at the Office for National Statistics ('ONS'). Mid-year population estimates by individual age are produced for each of the individual countries in the UK.

Death data for England & Wales are produced by the ONS Mortality Statistics department. The ONS includes all those deaths that occurred in England & Wales, whether or not the individual was a resident of England or Wales. This means that foreign nationals who died in England & Wales would be included in the death data, but that residents of England & Wales who died in a foreign country would not be included.

Deaths can be grouped either according to registration or occurrence. Occurrence data relates to deaths that occurred in a particular calendar year even though some of the deaths included may be registered after the end of that year. Registration data relates to deaths that were registered in a particular calendar year and will exclude deaths that have occurred but not yet been reported by the end of that year.

### *2.2 Data availability and limitations*

Data is available to download from the ONS website, but data is generally not available by single year of age for each calendar year. Data by single year of age has to be requested directly from the ONS, and there are restrictions over its use and subsequent distribution.

Members of the LLMA obtained a combined dataset in December 2011 of death registrations and mid-year population estimates by single year of age for different calendar years. The dataset consisted of ages 20 to 84 for both males and females for calendar years 1961 to 1970 and of ages 20 to 89 for both males and females for calendar years 1971 to 2010.

### *2.3 Calculation of crude central rates of mortality*

We have calculated crude central rates of mortality from mid-year population estimates and numbers of deaths for both males and females for calendar years 1961 to 2010 for the ages specified above. We have published these rates to 6 decimal places throughout.

# 3. LLMA longevity index for Germany

### *3.1 Source of population exposures and death data*

Between 1945 and 1990 the Statistisches Bundesamt ('SB') was the statistical office for the Bundesrepublik Deutschland (Federal Republic of Germany) and the Statistisches Zentralamt (Central Statistics Office) was the statistical office for the Deutsche Demokratische Republik (German Democratic Republic). Since 1990, the Statistisches Bundesamt ('SB') has been responsible for producing demographic statistics for the whole of Germany. The LLMA has used population estimates and death data for the whole of Germany for the historical longevity indices.

The SB's activities are prescribed in Federal Statistics Law. These include continuously collecting, compiling, processing, presenting and analysing data, such as population estimates and death data. In the case of population estimates and death data, the population that is counted and the frequency of data collection are therefore specified in legal terms either through the European Union or by a Federal law. This ensures a more consistent approach over time.

The SB uses census information as the basis for calculating population estimates. The most recent censuses were in 1987 and 2011. Census data are projected forward using death and net migration information in the intercensal period, and the SB conducted a census test in 2001 to determine whether the population estimates remained sufficiently accurate.

Migration data is available split by age from 1955, and split by both age and year of birth from 2000. This information is based on changes in registration data and should be reasonably accurate as individuals that wish to live in an area need to register at the local civil registrations office.

Death data are produced in respect of all people whose deaths were registered in Germany, with the exception of non-resident foreigners. German citizens who die abroad will be included in the deaths data if their deaths are registered in Germany.

### *3.2 Data availability and limitations*

The SB produces population estimates at both the end of each calendar year and at mid-year. The mid-year population estimates are calculated as an average of estimates for each of the 12 months through the year.

Death data are available for each calendar year split by sex, age at death and year of birth for ages 0–99 and 100+.

Death data will be understated to the extent that some Germans will have died abroad and their deaths were not registered in Germany for inheritance purposes. Death data are also adjusted each leap year by a factor of 365/366.

### *3.3 Calculation of crude central rates of mortality*

The Lifemetrics Advisory Committee had chosen to use the average of two consecutive end-of-year population estimates as the basis for mid-year population estimates. The LLMA has chosen to adopt the same approach. This enables us to calculate central rates of mortality directly.

The SB also provides migration data, and so an explicit adjustment for migration could have been made to the mid-year population estimate. This was not regarded as necessary because of the following factors:

- Migration data was only available from 1955.
- There are concerns over the migration data pre-1990 given inconsistencies over how migration flows were captured between the Federal Republic of Germany and the German Democratic Republic.
- The use of an average of end-year population estimates makes an implicit allowance for migration over the year.

A number of concerns were identified in the Lifemetrics Index over data quality for ages over 87 over the period before 1964 and for ages over 90 for later years.

We have therefore calculated crude central rates of mortality from mid-year population estimates and numbers of deaths for both males and females for ages 20 to 87 in calendar years 1956 to 1963 and for ages 20 to 90 in calendar years 1964 to 2010. We have published these rates to 6 decimal places throughout.

# 4. LLMA longevity index for Netherlands

### *4.1 Source of population exposures and death data*

The Netherlands has been producing and collecting demographic statistics since the mid 1800s, and a number of steps have been taken to improve the quality of such data, including issuing general guidelines in 1861 and obliging people to provide required information to local authorities.

The Gemeentelijke Basis Administratie persoonsgegevens (GBA system) was introduced on 1 October 1994 and is an automated municipal register. Every municipality has its own population register containing personal information on all non-resident individuals in that municipality. Access to these registers is only granted by the Ministry for the Interior.

The Centraal Bureau voor de Statistiek (CBS) has been granted permission by the Ministry for the Interior to receive from the municipal registers the data required to calculate various demographic statuses.

The CBS extracts information on the whole of the population once a year (1 January) and death data on a continuous basis. Data are collated and published centrally by a government agency. Prins (2000) contains a detailed discussion of how the information is generated through the GBA system, using different authorisations that have been granted to the CBS.

Deaths are declared to the local register where the death has taken place. In addition, before a person can be cremated or buried, cause-of-death information needs to be reported to a coroner. Death data and population estimates only consider Dutch citizens who are registered in the municipality. Tourists and Dutch citizens living abroad are excluded because they are not registered in any municipality.

Annual death data are available by individual year of age and gender according to two definitions of age – the 'exact age' at death and age at 31$^{st}$ December. 'Exact age' is defined as the age in whole years the individual attained on the day of dying, i.e., their age last birthday.

In contrast, 'age at 31$^{st}$ December' is defined as the age in whole numbers of years that a person would be on the last day of the calendar year, and is simply calculated by subtracting the year of birth from the year of death.

## *4.2 Data availability and limitations*

The population at the beginning of each calendar year (1 January) is available on the CBS website in Dutch, and historical data sets for individual ages 0–98 and collective older ages are available over the period 1950 to 2011.

Deaths of Dutch citizens who were out of Netherlands at the time of their death are usually registered for the purposes of inheritance. Dutch society relies heavily on its network of registrations for the provision of several benefits, so that the number of people who are unregistered is likely to be small.

## *4.3 Calculation of central rates of mortality*

Whilst not readily available on the CBS website, the CBS can produce special extractions that segment deaths data not only by calendar year and attained age but also by year of birth. It would therefore be possible to construct ‘cohort’ mortality rates that considered the deaths for common years of birth over adjacent calendar year periods. This is the approach that was recommended by the Lifemetrics Advisory Committee and is similar to the approach adopted for actuarial life tables for the Netherlands.

However, for consistency with the other country-specific longevity indices described in this Technical Manual, the LLMA has chosen to calculate central rates of mortality for a specified age and calendar year period for Netherlands. In a similar process to that for Germany, we used averages between population estimates at the start of adjacent years to produce the necessary mid-year population estimates, and we used the annual death data that was grouped by “exact age”.

We have calculated crude central rates of mortality on the LLMA website for both males and females for ages 20 to 90 for the period 1951 to 1994 and ages 20 to 98 for the period 1995 to 2010. We have published these rates to 6 decimal places throughout.

# 5. LLMA longevity index for USA

## *5.1 Source of population exposures and death data*

For the other countries we have considered, different departments in the national statistical office are responsible for the production of population estimates and deaths data. In the USA, death data are compiled by the Centers for Disease Control and Prevention ('CDC') and the National Center for Health Statistics ('NCHS'), and national population estimates are published by the US Census Bureau.

Annual population estimates are developed by the US Census Bureau as part of the Population Estimates Program (www.census.gov). This program produces total resident estimates of population on a monthly basis using the decennial census as a starting point. The US census has occurred every 10 years since 1790.

Intercensal estimates are constructed based on statistics on births, deaths, net immigration and net movement of military personnel collected by the CDC and the Department of Defence. The reference date for mid-year estimates is 1 July, and with each publication the estimates for all previously released years are revised.

For calendar years prior to 1980 the only set of population estimates that were available included both the US resident population and Armed Forces overseas. For later calendar years separate population estimates were produced that either included or excluded the Armed Forces overseas. We have used population estimates for the US resident population only where these were available.

Death registration data have been collected for the entire USA since 1933. Before then, death data was only published in respect of selected states or cities. Death registrations are collected from the 50 States and the District of Columbia and are available with classifications such as geographic area, age, sex and race.

Before 1970, resident mortality statistics included all deaths occurring in the USA, including those of non-residents, which were coded according to place of death. For 1970 and later, deaths of non-residents were excluded from US resident mortality data.

## 5.2 *Data availability and limitations*

Although death data are available for download from the CDC website, such data are not available by single year of age. Instead, the National Bureau of Economic Research hosts the Vital Statistics NCHS's Multiple Cause of Death database on its website. This provides mortality data in respect of calendar years 1959 to 2009, and the files are a listing of each registered death, along with information such as place of residence, place of occurrence, age, sex, race, cause of death and education. There is a much longer time lag in the release of US deaths data than the other countries that we have considered in this Technical Manual.

Death data are expected to be quite complete because all States have adopted laws requiring the registration of deaths. Small sources of errors would relate to coding errors, key entry errors, processing errors, unknown ages and misreporting of ages.

Significant amounts of population information are readily available on the website for the US Census Bureau (www.census.gov). Data are available at national, state and county levels, and data is segmented by age, gender and race.

Sources of errors in the post-censal estimates are generally the result of miscounting or from misreporting of age. These errors are often more serious for some subsets of the population, and so for example in the review of the 1990 census the female population was more completely counted than the male population and the white population was more completely counted than the black population. In terms of age groups, the least accurate were those aged 85 and over.

## 5.3 *Calculation of crude central rates of mortality*

We have calculated crude central rates of mortality from mid-year population estimates and numbers of deaths for both males and females for ages 20 to 84 for calendar years 1968 to 2009. We have published central rates of mortality to 6 decimal places throughout.

# 6. Graduation methodology

### *6.1 Calculation of graduated mortality rates*

Graduated mortality rates are constructed from crude mortality rates using a smoothing, or “graduation” procedure.

Graduation of mortality rates is desirable to reduce noise and is justified because mortality rates for adjacent ages are similar and highly correlated. Graduation captures additional information about the mortality rate at a particular age from the rates at nearby ages. As a result, graduation does not destroy the integrity of the data, but instead brings the twin benefits of simplification and noise reduction. Graduated mortality rates are important in the valuation of longevity and mortality exposures and the calculation of life expectancy.

For each calendar year that we have published historical crude central rates of mortality for the different country-specific longevity indices, we calculated graduated central rates of mortality over the same selected age range, with the graduation for each calendar year being carried out independently of other graduations.

The UK Government Actuary’s Department has adopted cubic splines in the construction of more recent English Life Tables. A spline is a piecewise polynomial function that is differentiable a defined number of times, namely the degree of the polynomial minus one. A spline is therefore defined by the locations of the knots and the coefficients of the polynomials which define the function on each interval.

We placed knots at every 5 years within the selected age range, and estimated the coefficients of the various cubic basis splines by minimising the following function:

$$p\sum_{x=x_0}^{x_1}(\tilde{m}_x-\tilde{s}_x)^2+\ (1-p)\sum_{x=x_0+1}^{x_1-1}(\tilde{s}_{x-1}-2\tilde{s}_x+\tilde{s}_{x+1})^2$$

where $\tilde{m}_x$ is log (crude central rates of mortality for age x), $\tilde{s}_x$ is log (graduated central rates of mortality for age x), $x_0$ and $x_1$ are the minimum and maximum ages in the selected age range and we have chosen p, the smoothing constant, to be equal to a value of 1/3.

The first term represents goodness of fit through mean squared differences between graduated central rates of mortality and crude central rates of mortality. The second term is a penalty term which increases with fluctuations of the polynomial. The trade-off between smoothness and goodness of fit is controlled by the value of p, and a higher value of p indicates a greater emphasis on goodness of fit.

We used the actuarial approximation in section **1.3.4** to calculate graduated initial rates of mortality from the graduated central rates of mortality.

## 6.2 Higher age methodology

The estimation of mortality rates at high ages is subject to larger errors than at younger ages due to:

- Inaccurate or unavailable birth records
- Tendency for certain individuals of a high age to exaggerate their age
- Inaccurate reporting of age at death
- Small populations of individuals at higher ages means that noise in the data is greater than at lower ages

In the absence of credible data, a higher age methodology is needed to produce suitable mortality rates at higher ages so that life expectancies can be calculated and various products such as life assurance, pensions and annuities can be valued.

For the higher age methodology, we have chosen a parametric approach similar to that adopted in Lifemetrics, with a number of methodologies having been suggested by members of the IOC and various tests having been examined as to the suitability and application of such methodologies.

Our chosen approach uses least-squares to fit a straight line through log *(graduated central rate of mortality)* for the ten oldest ages in the selected age range for each calendar year. The graduated and fitted lines are blended over this selected age range. A cubic function is then used to extrapolate initial rates of mortality to age 119, matching both value and slope of the initial rate of mortality at the maximum age in the selected age range and a country-specific initial rate of mortality assumptions at age 119.

These initial rates of mortality at 119 are as for the Lifemetrics Index and were based on mortality rates in published actuarial tables in the different countries:

- England & Wales: 60%
- Germany: 60%
- Netherlands 80%
- USA 50%

The slope of the initial rate of mortality at age 119 was assumed to be zero and the value of the initial rate of mortality at age 120 was assumed to be 1 for all longevity indices.

We used R code both to produce graduated initial rates of mortality over the selected age ranges and to produce initial rates of mortality at higher ages through the use of a suitable cubic function.

We have calculated initial rates of mortality for both males and females for ages 20 to 90 for each selected calendar year for the different country-specific longevity indices, except for calendar years 1995 to 2010 for the Netherlands where initial rates of mortality were published for ages 20 to 98. All initial rates of mortality were published to 6 decimal points.

### *6.3 Calculation of complete life expectancy*

The LLMA has further published complete period life expectancies for all longevity indices as a metric for remaining life expectancy at different ages.

The Lifemetrics Index published curtate period life expectancies, $e_x$, which can be calculated directly from initial rates of mortality in a single calendar year, assuming no future change in mortality rates.

$$e_x = \sum_{i=1}^{120-x} \left( \prod_{j=0}^{i-1} (1 - q_{x+j}) \right)$$

This produces the expected number of complete years lived. However, on average individuals would be expected to live half a year beyond their final birthday, and so complete period life expectancy is equal to the curtate period life expectancy plus half a year. We have published complete period life expectancies to 2 decimal points, based on unrounded initial rates of mortality.

# References

Coughlan, G.D., Epstein, D., Ong, A., Sinha, A., Balevich, I., Hevia-Portocarrera, J., Gingrich, E., Khalaf Allah, M. & Joseph, P. (2007). LifeMetrics: A toolkit for measuring and managing longevity and mortality risks. Technical Document (JPMorgan: London, March 13, 2007).

Coughlan, G.D., Epstein, Hevia-Portocarrera, J., Khalaf Allah, M., Watts, C.S., Joseph, P. (2007). LifeMetrics: Netherlands Longevity Index. Technical Document Supplement (JPMorgan: London, October 22, 2007).

Coughlan, G.D., Epstein, D., Watts, C., Khalaf-Allah, M., Joseph, P., Ye, Y. (2008). Lifemetrics: German Longevity Index. Technical Document Supplement (JPMorgan: London, April 7, 2008).

Prins, C.J.M. (2000). Dutch population statistics based on population register data. Maandstatistiek van de bevolking 48 (2), 9–15.

www.llma.org