# Technical Note

## The S-forward

(Life & Longevity Markets Association)

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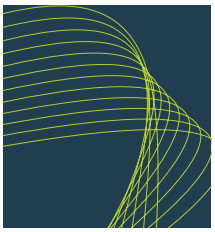


## 1. Introduction

The S-forward or 'Survivor' forward is a financial derivative that can be used to manage or actively take exposure to longevity risk. It is a cash settled contract linked to survival rates of a given population cohort which are ultimately derived from mortality rates. This derivative has also been referred to as the tpx-forward. '$_tp_x$' and 'S' are references to survival probability (or simply survival) to which the instrument facilitates exposure.

The S-forward is the basic building-block for the longevity (survivor) swaps that have already been used by pension funds and insurers to hedge longevity risk. These longevity swaps essentially comprise a stream of S-forwards with different maturity dates. Longevity survivor swaps were first described by Dowd (2003).

An S-forward term sheet proposed as a template for use in transactions has also been developed by the LLMA and is available separately – see *Sample Term Sheet: S-forward* (LLMA (2010a)). In addition, the LLMA has developed a spreadsheet to assist in the valuation of this instrument – see *S-forward Valuation Spreadsheet* (LLMA (2010b)).

## 2. Definition

An S-forward is an agreement between two counterparties to exchange at a future date (the maturity of the contract) an amount equal to the realized survival rate of a given population cohort, in return for a fixed survival rate agreed at the inception of the contract. The definition of a survival rate is in turn dependent on and calculated from mortality rates. For one year, the survival rate is simply one minus the probability of death occurring over the following year:

$$p_x(t) = 1 - q_x(t) \qquad (1)$$

where:

$q_x(t)$ is the 1-year initial mortality rate, for age group x at time t

$p_x(t)$ is the 1-year survival probability, for age group x at time t

The n-year survival rate for a person or group aged x today needs forward mortality rates for calculation. The survival probability for a person or population aged x in **n** years time must be dependent on the contingent probabilities that a person aged x today survives every year up to the end of year **n**. From basic probability theory, the survival probability (which we will call the 'survival rate') that a person aged x at time t = 0 survives to the end of year n) can be expressed as $_nP_x$ , where:

$$_np_x = \prod_{i=0}^{n-1}[1 - q_{x+i}(t_i)] \qquad (2)$$

So for example, for a 65 year old today, surviving for a further three years has the associated rate:

$$ {}_3p_{65} = [1 - q_{65}(0)] \times [1 - q_{66}(1)] \times [1 - q_{67}(2)] \qquad (3) $$

The S-forward instrument is a zero-coupon fixed-for-floating survival swap, which is effectively a single exchange of payments at maturity of the instrument. Notation for the S-forward will take the form ${}_tS_x$ , where the survival forward references the population cohort currently aged x in year t.

The floating leg of the instrument references the uncertain future mortality rates, and thus the uncertain survival rate, of a given population cohort as reflected by an appropriate index over the observation period of the contract. The fixed leg is the fixed rate a counterparty to the transaction would wish to receive for committing to pay the floating leg of the transaction.

The flows of the transaction are illustrated graphically in Exhibit 1. The reference rate for settling the floating leg of the contract is the realized survival rate (based on realized mortality) as determined for an appropriate index over the observation period, as specified at inception of the transaction.

In an S-forward transaction, a longevity protection buyer would be the Fixed Rate Payer, whilst a protection seller would be the Floating Rate Payer.

Such an S-forward transaction may be between a life insurance provider (a protection seller) and a pension fund or annuity provider (a protection buyer). If the actual realized mortality is less than expected (i.e. the realized survival rate, or equivalently longevity, is higher than initially expected, as reflected by the fixed rate) then the longevity protection buyer receives a net payment, which compensates them for the increase in their longevity-linked liability.

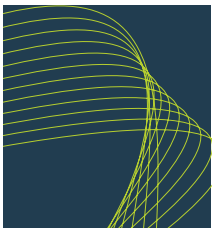
## 3. Floating rate payment

The floating rate payment used to settle the S-forward will be determined by the realized value of mortality rates (the realized survival rate) for the index underlying the contract over the observation period, each as specified at the transaction inception.

The value of the index used to determine the floating rate payment is typically calculated by an independent Index Calculation Agent – not to be confused with the Calculation Agent for the transaction, which may be one of the counterparties to the transaction.

## 4. Fixed rate payment

The fixed rate payment is agreed at the time of trading by the parties to the transaction. It is essentially the estimate of the survival rate for the given population cohort over the observation period specified in the transaction, plus or minus bid/offer costs for entering into the transaction. The mid forward survival rate for the observation period will likely be different from the best estimate of the survival rate for the period predicted by mortality tables. The difference between survival rates predicted by mortality tables and the parties entering into the transaction is reflective of differing mortality assumptions the parties to the transaction assume to those used to calculate mortality tables. This difference is illustrated in Exhibit 2 as the difference between Expected Survival rates (determined from base mortality tables), and Expected Survival rates with Risk Premium, and is referred to as the risk premium.



## 5. Cash flows

The only cash flows exchanged in an S-forward are at maturity. Contract settlement allows for netting, so that at only one of the counterparties owes payment at maturity.

Consider an example in which a longevity protection buyer pays the fixed rate and receives the floating rate on an S-forward contract with maturity *T*. The fixed rate for the transaction is the forward survival rate agreed between the parties for the index at time t = 0 corresponding to the forward time period *T* and is denoted by $p_{\text{forward}}(0{:}T)$. The realised survival rate (floating rate) for the index at maturity T is denoted by $p_{\text{realized}}(0{:}T)$. The net payment that takes place at maturity T, the Net Payoff Amount (*NPA*) is given by:

$$NPA(T) = Notional \times [\, p_{realized}(0{:}T) - p_{forward}(0{:}T)] \qquad (4)$$

If NPA(T) is a positive number, this cashflow is an amount receivable by the longevity protection buyer. If NPA(T) is a negative number, this is an amount payable by the longevity protection buyer.

It is seen that the fixed rate payer (protection buyer) has a long longevity position, since the fixed rate payer benefits if realized survival rates rise above expectations (i.e., mortality falls below expectations). By contrast a floating rate payer (protection seller) would receive the fixed rate and pay the floating rate (realized survival rate), benefiting if the realised survival rates fall below expectations (i.e., mortality rises above expectations).

## 6. References


LLMA (2010a) *Sample Term Sheet: S-forward* [www.llma.org]

LLMA (2010b) *S-forward Valuation Spreadsheet* [www.llma.org]

Dowd, K. (2003). "Survivor Bonds: A Comment on Blake and Burrows." *Journal of Risk and Insurance,* 2003, Vol. 70, No. 2, pp. 339-348..

***Exhibit 1:*** *Cash flows associated with an S-forward contract.*

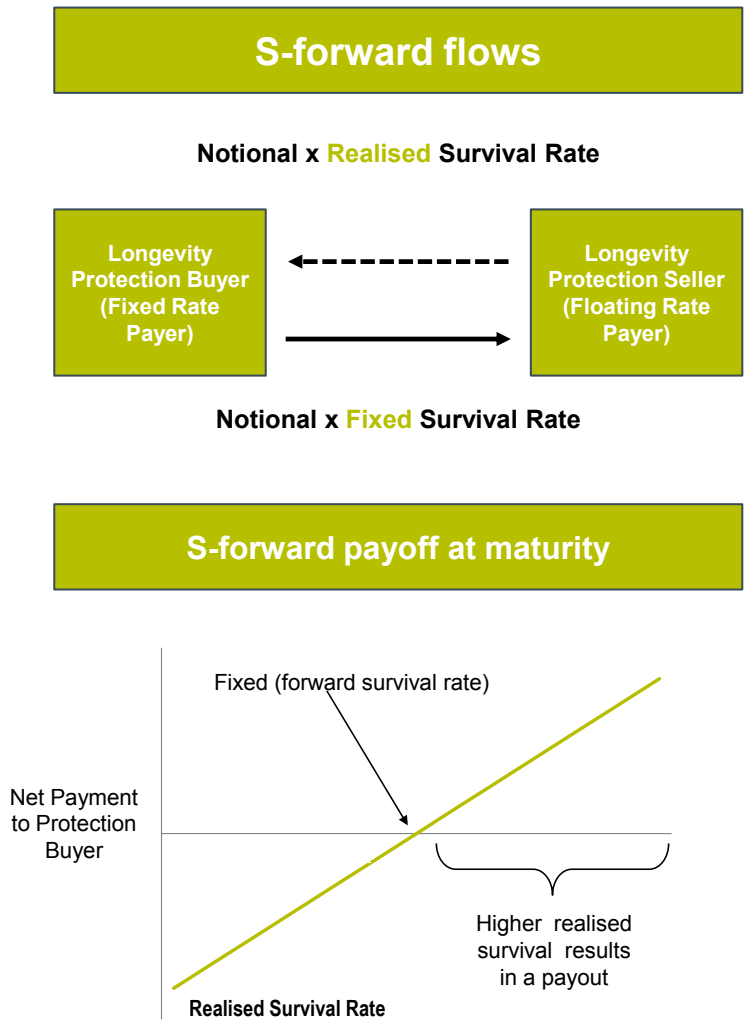
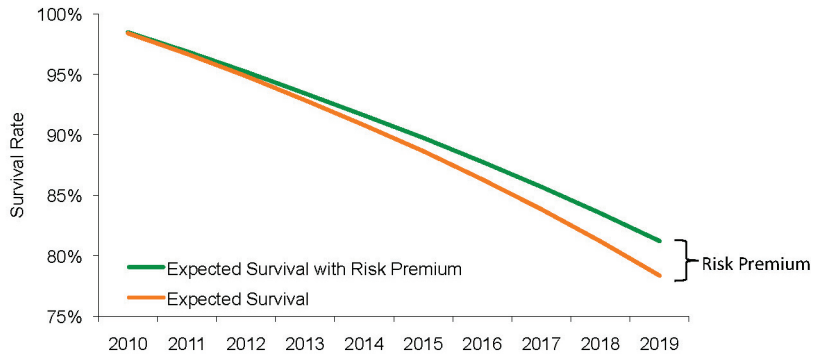


***Exhibit 2:*** *Valuation and pricing of an S-forward contract*



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